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Magnetic Field and Inductance Calculations in Theta-Pinch and Z-Pinch Geometries

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Two codes have been developed to model solid metal or wire-wound conductors. The calculations are based on the decomposition of the conductors into arrays of thin wires. The first code, EDDY, models cylindrically symmetric conductors with currents in the theta direction. This code accurately models eddy current induction and magnetic diffusion. It was created in order to aid the design of magnetic-field shields in the FRX-L experiment for Magnetized Target Fusion (MTF). EDDY uses fast, accurate elliptic integral subroutines from MATLAB to solve for the time-dependent current flowing through each wire loop and the resultant magnetic field configuration. The second code, INDIV, models arbitrarily shaped conductors with current flow in the *z* direction. It was designed to model current division in an inductive divider that would inject current into a liner cavity, for magnetic flux and magnetized-plasma compression experiments. An experiment has been performed to test the INDIV code and the inductive division concept. The numerical results compare well with those of the experiment.

KEY WORDS: Eddy current induction; magnetic diffusion; shielding; Magnetized Target Fusion.

INTRODUCTION

Two codes have recently been developed at the University of Nevada, Reno to model solid metal or wire-wound conductors. The calculations are based on the decomposition of the conductors into arrays of thin wires. The first code, EDDY, models cylindrically symmetric conductors with theta currents. The second code, INDIV, calculates fields and inductances for conductors in z-pinch geometry. These two codes are discussed, in turn.

EDDY *R-Z* Code

EDDY is a two-dimensional $R-Z$ code that calculates eddy current induction, resistive diffusion, and the resultant magnetic field of cylindrically symmetric conductors. EDDY models cylindrically symmetric solid metal objects with tightly packed arrays of circular wire loops, each with circular cross section. The code uses fast, accurate elliptic integral subroutines from MATLAB to solve for the time-dependent current flowing through each wire loop and the resultant magnetic field configuration.

Eddy *R-Z Code Performance*

Before calculating magnetic properties of conductors in complex geometries, we performed simple calculations in geometries with analytic solutions. We first tested the static capabilities of the code by modeling the field produced by a long solenoid. Next, we modeled two wires in close proximity to one another, where the first loop was driven with an oscillating current, and the second loop was inductively coupled to it. For both test calculations, the

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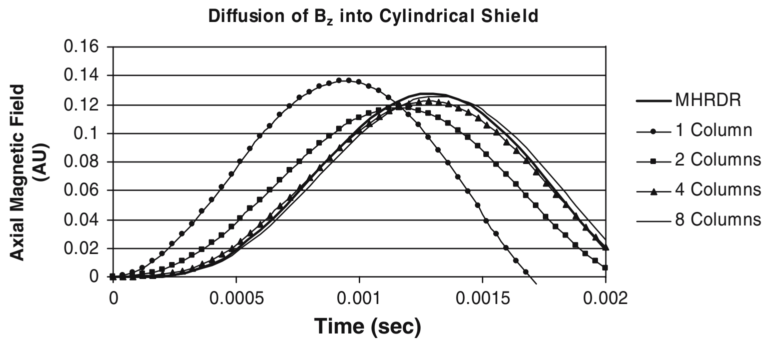

Fig. 1. Diffusion of an axial magnetic field into cylindrical conductor. With the cylindrical radial thickness decomposed into 4 or more wires, the EDDY results are close to those of the Eulerian MHD code MHRDR.

code matched analytic solutions very closely. Next, we modeled the diffusion of an axial magnetic field B_z into a coaxial cylindrical conductor. We first tested code convergence by increasing the number of radial columns composing the shield. We then compared the solutions with results from MHRDR, a benchmarked Eulerian MHD code. The diffusion fields from the two codes are plotted in Figure 1. When using at least four columns the codes agree very well.

Calculations Relevant to Plasma Formation and Translation for MTF

The EDDY code was designed to calculate currents and fields in a geometry relevant to the formation, translation, and compression of a field-reversed-configuration (FRC) plasma. Generic features of such a system include a fast rising, high voltage conical theta coil for plasma formation and initial axial acceleration, a translation region with mirrored ends consisting of slowly rising, low voltage, multi-turn coils, and various conductors used for magnetic shielding. Here we focus on the EDDY code's ability to predict voltage transients that occur in pulsed high field environments. Specifically, the code is being used by members of the P-24 Plasma Physics group at Los Alamos National Laboratory as a design tool for magnetic coils and shielding in the FRX-L experiment. In order to characterize and optimize the FRC plasma, the plasma must be held axially stationary; mirror fields are used to accomplish this. Figure 2 shows calculations of magnetic flux contours and magnetic pressures for a specified end-mirror geometry.

INDIV R-θ CODE FOR FIELD AND INDUCTANCE CALCULATIONS IN NON-SYMMETRIC Z-PINCH GEOMETRIES

The second code, INDIV, models conductors in a z-pinch geometry. The code was written primarily as a tool to calculate inductances and current splitting in quasi-symmetric power flow geometries. This is of particular interest for experiments in which magnetic flux or magnetized plasma is compressed by an imploding liner.

In particular, INDIV was developed to calculate the current division in an "inductive divider" used to divert a portion of the main bank current onto a

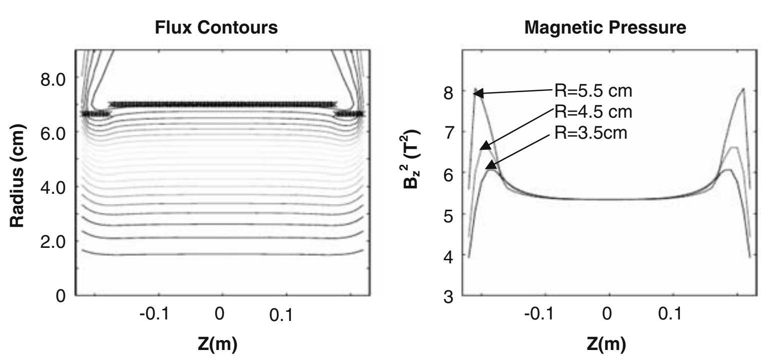

Fig. 2. Flux contours and magnetic pressure calculated by EDDY for an FRC plasma translation system.

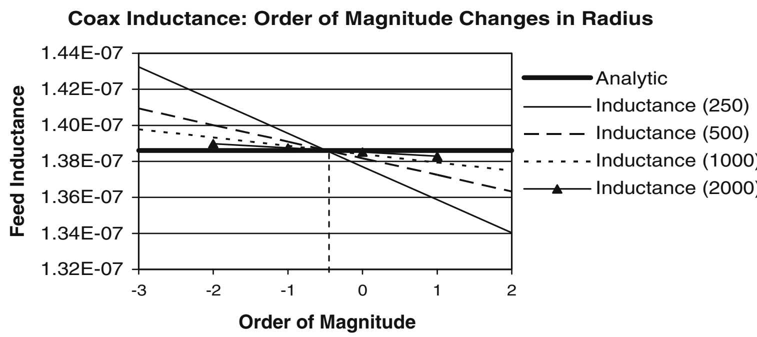

Fig. 3. Inductance vs. filament radius, to determine the optimum wire spacing for INDIV calculations. Zero on the order-of-magnitude axis corresponds to the filament radius r_0 at which the filaments are as tightly packed as possible without overlap.

conducting hard core inside a liner [1]. The inductive divider creates two current paths. First, current flows along a central cylinder which feeds the liner cavity. This cylinder is surrounded by a set of rods or "shunt" inductors, which carry current directly to the outside of the liner. The core current loops back on the inside of the liner, and then recombines with the main current on the outside of the liner, so it does not impede the liner implosion. The code calculates the shunt inductance, the mutual inductance between the shunt inductors and the hard core, and the resultant current division of the system.

All the conductors have thickness greater than one skin depth, and thus do not carry current uniformly. We therefore model both cylinders and rods with many constituent conductors to account for non-uniform current distribution.

Determination of Optimum Wire Spacing for INDIV Accuracy

In a code where solid conductors are modeled in a filamentary manner, it is important to determine which wire spacing gives the best results. In order to determine this, we model coaxial cylinders with known inductance. By increasing the number of constituent wires of the cylinders, and then varying the radius of the filaments by factors of ten we were able to optimize the calculation accuracy (Figure 3).

Figure 3 shows that the larger the number of wires, the lower the sensitivity to changes in wire radius (smaller slope). More important, all curves intersect the analytic solution with the same order of magnitude change in radius. The radius to be used is thus

$$r = r_o \times 10^{-0.5} = r_o \Big/ \sqrt{10} \qquad (1)$$

where r_0 is defined to be the cross sectional radius where the wires touch but do not overlap.

Experimental Verification of INDIV Performance

In order to check the viability of the current divider concept, as well as gauge the INDIV code performance, a small pulsed-power device and a prototypical inductive current divider assembly were

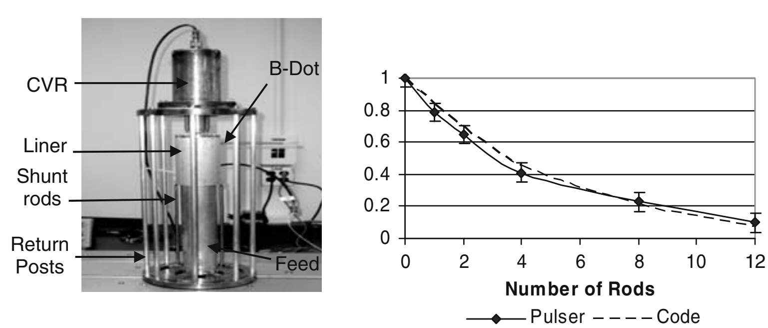

Fig. 4. Experimental verification of the INDIV R-θ code. Left: Inductive current divider driven by a 12-kA pulser (not shown below). Right: experimentally measured and calculated current ratio (current through liner cavity/total current), as a function of the number of shunt rods.

constructed. The number of shunt posts in the assembly can be altered from 0 to 12. The current inside the flux chamber was measured with an internal B-dot probe, and the total current was measured with a current viewing resistor (CVR). Figure 4 shows the divider assembly alongside a plot that compares experimental data with the current division predicted by INDIV.

CONCLUSION

Two codes have been developed and benchmarked to model solid metal or wire-wound conductors. The electromagnetic properties of solid conductors can be accurately modeled with tightly packed wire arrays. The EDDY R-Z code calculates eddy current induction, magnetic field diffusion, and magnetic fields from azimuthally symmetric conductors. The INDIV R-θ code calculates inductances and current division in inductive dividers with axial symmetry.

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